List of definitions for Midterm 2

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1 f has an absolute maximum/minimum

Let c be a number in the domain D of a function f. Then $f(c)$ is the:

- **absolute maximum** value of f on D if $f(c) \geq f(x)$ for all x in D
- **absolute minimum** value of f on D if $f(c) \leq f(x)$ for all x in D

2 f has a local maximum/minimum

The number $f(c)$ is a:

- **local maximum** value of f if $f(c) \geq f(x)$ when x is near c
- **local minimum** value of f if $f(c) \leq f(x)$ when x is near c

3 The Extreme Value Theorem

Extreme Value Theorem: If f is continuous on a closed interval $[a, b]$ then f attains an absolute maximum value $f(c)$ and an absolute minimum value $f(d)$ at some numbers c and d in $[a, b]$.

4 Rolle's Theorem

Rolle's Theorem: Let f be a function that satisfies the following three hypotheses:

1) f is continuous on the closed interval $[a, b]$
2) f is differentiable on the open interval (a, b)
3) $f(a) = f(b)$

Then there is a number c in (a, b) such that $f'(c) = 0$.

Note: Think of the example of the upper semi-circle $y = \sqrt{1 - x^2}$ to help you memorize the hypotheses of the theorem!

5 Mean Value Theorem

Mean Value Theorem: Let f be a function that satisfies the following two hypotheses:

1) f is continuous on the closed interval $[a, b]$
2) f is differentiable on the open interval (a, b)

Then there is a number c in (a, b) such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

or, equivalently,

$$f(b) - f(a) = f'(c)(b - a)$$

6 Uniqueness of antiderivatives

Theorem: If $f'(x) = 0$ for all x in an interval (a, b), then f is constant on (a, b)

Corollary: If $f'(x) = g'(x)$ for all x in an interval (a, b), then $f - g$ is constant on (a, b); that is, $f(x) = g(x) + C$ where C is a constant.

7 Proof of the above theorem

1) Let x_1 and x_2 be any two numbers in (a, b) with $x_1 < x_2$

2) Since f is differentiable on (a, b), it must be differentiable on (x_1, x_2) and continuous on $[x_1, x_2]$.

3) By applying the **Mean Value Theorem** to f on the interval $[x_1, x_2]$, we get a number c such that $x_1 < c < x_2$ and:

$$f(x_2) - f(x_1) = f'(c)(x_2 - x_1)$$

4) Since $f'(x) = 0$ for all x, we have $f'(c) = 0$, and so the above equation becomes:

$$f(x_2) - f(x_1) = 0 \qquad \text{or} \qquad f(x_2) = f(x_1)$$

5) Therefore, f has the same value at *any* two numbers x_1 and x_2 in (a, b).
This means that f is constant on (a, b)

Test yourself!

Now, without looking at the definitions on the previous pages, try to define the following terms. Then compare your answers to the definitions above, and correct any mistake you make. You have to memorize those definitions **word by word**, e-mail me if you have any doubts about a definition!

1. If $f'(x) = 0$ for all x in an interval (a, b), what can you conclude? **Prove this!**
2. Uniqueness of Antiderivatives
3. f has a global minimum
4. f has a local maximum
5. The Extreme Value Theorem
6. Rolle's Theorem
7. The Mean Value Theorem